

4726 Further Pure Mathematics 2

- 1 (i) Give $1 + 2x + (2x)^2/2$
Get $1 + 2x + 2x^2$
- M1 Reasonable 3 term attempt e.g. allow $2x^2/2$
A1 cao
SC Reasonable attempt at $f'(0)$ and $f''(0)$ M1
Get $1+2x+2x^2$ cao A1
- (ii) $\ln((1+2x+2x^2) + (1-2x+2x^2)) =$
 $\ln(2+4x^2) =$
 $\ln 2 + \ln(1 + 2x^2)$
 $\ln 2 + 2x^2$
- M1 Attempt to sub for e^{2x} and e^{-2x}
A1√ On their part (i)
M1 Use of log law in reasonable expression
A1 cao
SC Use of Maclaurin for $f'(x)$ and $f''(x)$ M1
One correct A1
Attempt $f(0)$, $f'(0)$ and $f''(0)$ M1
Get cao A1
- 2 (i) $x_2 = 1.8913115$
 $x_3 = 1.8915831$
 $x_4 = 1.8915746$
- B1 x_2 correct; allow answers which round
B1√ For any other from their working
B1 For all three correct
- (ii) $e_3/e_2 = -0.031(1)$
 $e_4/e_3 = -0.036(5)$
State $f'(a) \approx e_3/e_2 \approx e_4/e_3$
- M1 Subtraction and division on their values;
allow \pm
A1 Or answers which round to -0.031 and -0.037
B1√ Using their values but only if approx. equal;
allow differentiation if correct conclusion;
allow gradient for f'
- 3 (i) Diff. $\sin y = x$
Use $\sin^2 + \cos^2 = 1$ to A.G.
Justify +
- M1 Implicit diff. to $dy/dx = \pm(1/\cos y)$
A1 Clearly derived; ignore \pm
B1 e.g graph/ principal values
- (ii) Get $2/(\sqrt{1-4x^2}) + 1/(\sqrt{1-y^2}) dy/dx = 0$
Find $y = \sqrt{3}/2$
Get $-2\sqrt{3}/3$
- M1 Attempt implicit diff. and chain rule; allow e.g. $(1-2x^2)$ or $a/\sqrt{1-4x^2}$
A1
M1 Method leading to y
A1√ AEEF; from their a above
SC Write $\sin(\frac{1}{2}\pi - \sin^{-1}2x) = \cos(\sin^{-1}2x)$ B1
Attempt to diff. as above M1
Replace x in reasonable dy/dx and attempt to tidy M1
Get result above A1

8 (i)	Area = $\int 1/(x+1) dx$	B1	Include or imply correct limits
	Use limits to $\ln(n+1)$	B1	
	Compare area under curve to areas of rectangles	B1	Justify inequality
	Sum of areas = $1x(1/2 + 1/3 + \dots + 1/(n+1))$	M1	Sum seen or implied as $1 \times y$ values
	Clear detail to A.G.	A1	Explanation required e.g. area of last rectangle at $x=n$, area under curve to $x=n$
(ii)	Show or explain areas of rectangles above curve	M1	
	Areas of rectangles (as above) > area under curve	A1	First and last heights seen or implied; A.G.
(iii)	Add 1 to both sides in (i) to make $\sum(1/r)$	B1	Must be clear addition
	Add $1/(n+1)$ to both sides in (ii) to make $\sum(1/r)$	B1	Must be clear addition; A.G.
(iv)	State divergent	B1	Allow not convergent
	Explain e.g. $\ln(n+1) \rightarrow \infty$ as $n \rightarrow \infty$	B1	
9 (i)	Require denom. = 0	B1	
	<u>Explain</u> why denom. $\neq 0$	B1	Attempt to solve, explain always > 0 etc.
(ii)	Set up quadratic in x	M1	
	Get $2yx^2 - 4x + (2a^2y + 3a) = 0$	A1	
	Use $b^2 \geq 4ac$ for real x	M1	Produce quadratic inequality in y from their quad.; allow use of = or <
	Attempt to solve their inequality	M1	Factors or formula
	Get $y > 1/2a$ and $y < -2/a$	A1	Justified from graph
			SC Attempt diff. by quot./product rule M1
			Solve $dy/dx = 0$ for two values of x M1
			Get $x=2a$ and $x=-a/2$ A1
			Attempt to find two y values M1
			Get correct inequalities (graph used to justify them) A1
(iii)	Split into two separate integrals	M1	
	Get $k \ln(x^2 + a^2)$	A1	Or $p \ln(2x^2 + 2a^2)$
	Get $k_1 \tan^{-1}(x/a)$	A1	k_1 not involving a
	Use limits and attempt to simplify	M1	
	Get $\ln 2.5 - 1.5 \tan^{-1} 2 + 3\pi/8$	A1	
			AEEF
			SC Sub. $x = a \tan \theta$ and $dx = a \sec^2 \theta d\theta$ M1
			Reduce to $\int p \tan \theta - p_1 d\theta$ (ignore limits here) A1
			Integrate to $p \ln(\sec \theta) - p_1 \theta$ A1
			Use limits (old or new) and attempt to simplify M1
			Get answer above A1